Exercise 2.4 (Polynomials)

1. Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

(i) $2x^3+x^2-5x+2; \frac{1}{2}, 1, -2$

(ii) $x^3-4x^2+5x-2; 2, 1, 1$

Ans. (i) Comparing the given polynomial with ax^3+bx^2+cx+d
we get
a=2 ,b=1,c=-5 and d = 2.

$p\left(\frac{1}{2}\right)=2\left(\frac{1}{2}\right)^3+\left(\frac{1}{2}\right)^2-5\left(\frac{1}{2}\right)+2$

$=\frac{1}{4}+\frac{1}{4}-\frac{5}{2}+2$

$=\frac{1+1-10+8}{4}=0$

$p(1)=2(1)^3+(1)^2-5(1)+2$

$=2+1-5+2=0$

$p(-2)=2(-2)^3+(-2)^2-5(-2)+2$

$=2(-8)+4+10+2=-16+16=0$

$\therefore \frac{1}{2}, 1$ and -2 are the zeroes of $2x^3+x^2-5x+2$.

Now, $\alpha+\beta+\gamma$

$= \frac{1}{2}+1+(-2) = \frac{1+2-4}{2} = \frac{-1}{2} = \frac{-b}{a}$

And $\alpha\beta+\beta\gamma+\gamma\alpha$

$= \left(\frac{1}{2}\right)(1)+(1)(-2)+(-2)\left(\frac{1}{2}\right)$

$= \frac{1}{2}-2-1$ $= \frac{-5}{2} = \frac{c}{a}$

And $\alpha\beta\gamma$ $= \frac{1}{2}\times 1\times(-2) = -1 = \frac{-2}{2} = \frac{-d}{a}$

(ii) Comparing the given polynomial with ax^3+bx^2+cx+d, we get
$a=1, b=-4, c=5$ and $d=-2$.

$p(2) = 2(2)^3-4(2)^2+5(2)-2$

$= 8-16+10-2$ = 0

$p(1) = (1)^3-4(1)^2+5(1)-2$

$= 1-4+5-2$ = 0

2,1 and 1 the zeroes of $x^3 - 4x^2 + 5x - 2$

Now, $\alpha + \beta + \gamma$

$= 2 + 1 + 1 = 4 = \frac{-(-4)}{1} = \frac{-b}{a}$

And $\alpha\beta + \beta\gamma + \gamma\alpha = (2)(1) + (1)(1) + (1)(2)$

$= 2 + 1 + 2 = \frac{5}{1} = \frac{c}{a}$

And $\alpha\beta\gamma = 2 \times 1 \times 1 = 2 = \frac{-(-2)}{1} = \frac{-d}{a}$

2. Find a cubic polynomial with the sum of the product of its zeroes taken two at a time and the product of its zeroes are 2,-7 and -14 respectively.

Ans. Let the cubic polynomial be $ax^3 + bx^2 + cx + d$ and its zeroes be α, β and γ.

Then $\alpha + \beta + \gamma = 2 = \frac{-(-2)}{1} = \frac{-b}{a}$ and $\alpha\beta + \beta\gamma + \gamma\alpha = -7 = \frac{-7}{1} = \frac{c}{a}$

And $\alpha\beta\gamma = -14 = \frac{-14}{1} = \frac{d}{a}$

Here, $a = 1, b = -2, c = -7$ and $d = 14$

Hence, cubic polynomial will be $x^3 - 2x^2 - 7x + 14$.

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3. If the zeroes of the polynomial $x^3 - 3x^2 + x + 1$ are a-b , a,a+b . Find a and b.

Ans. Since $(a-b), a, (a+b)$ are the zeroes of the polynomial $x^3 - 3x^2 + 3x + 1$.

$\therefore \alpha + \beta + \gamma = a - b + b + a + b = \frac{-(-3)}{1} = 3$

$\Rightarrow 3a = 3$

$\Rightarrow a = 1$

And $\alpha\beta + \beta\gamma + \gamma\alpha$

$= (a-b)a + a(a+b) + (a+b)(a-b) = \frac{1}{1} = 1$

$\Rightarrow a^2 - ab + a^2 + ab + a^2 - b^2 = 1$

$\Rightarrow 3a^2 - b^2 = 1$

$\Rightarrow 3(1)^2 - b^2 = 1 \; [\because a = 1]$

$\Rightarrow 3 - b^2 = 1$

$\Rightarrow b = \pm 2$

Hence $a = 1$ and $b = \pm 2$.

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4. If the two zeroes of the polynomial $x^4-6x^3-26x^2+138x-35$ are $2\pm\sqrt{3}$, find other zeroes.

Ans. Since $2\pm\sqrt{3}$ are two zeroes of the polynomial $p(x)=x^4-6x^3-26x^2+138x-35$.

Let $x=2\pm\sqrt{3} \Rightarrow x-2=\pm\sqrt{3}$

Squaring both sides, $x^2-4x+4=3$

$\Rightarrow\ x^2-4x+1=0$

Now we divide $p(x)$ by x^2-4x+1 to obtain other zeroes.

$$
\begin{array}{r}
x^2-2x-35 \\
x^2-4x+1 \;)\; x^4-6x^3-26x^2+138x-35 \\
\pm x^4 \mp 4x^3 \pm x^2 \qquad\qquad\qquad \\
\hline
-2x^3-27x^2+138x \qquad \\
\mp 2x^3 \pm 8x^2 \mp 2x \qquad \\
\hline
-35x^2+140x-35 \\
\mp 35x^2 \pm 140x \mp 35 \\
\hline
0
\end{array}
$$

$\therefore p(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$

$= (x^2 - 4x + 1)(x^2 - 2x - 35)$

$= (x^2 - 4x + 1)(x^2 - 7x + 5x - 35)$

$= (x^2 - 4x + 1)[x(x-7) + 5(x-7)]$

$= (x^2 - 4x + 1)(x+5)(x-7)$

$\Rightarrow (x+5)$ and $(x-7)$ are the other factors of $p(x)$.

-5 and 7 are other zeroes of the given polynomial.

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5. If the polynomial $x^4-6x^3+16x^2-25x+10$ is divided by another polynomial x^2-2x+k, the remainder comes out to be $x+a$, find k and a.

Ans. Let us divide $x^4-6x^3+16x^2-25x+10$ by x^2-2x+k.

$$
\begin{array}{r|l}
 & x^2-4x+(8-k) \\
x^2-2x+k & x^4-6x^3+16x^2-25x+10 \\
 & \pm x^4 \mp 2x^3 \pm kx^2 \\ \hline
 & -4x^3+(16-k)x^2-25x+10 \\
 & \mp 4x^3 \pm 8x^2 \mp 4kx \\ \hline
 & (8-k)\ x^2+(4k-25)x+10 \\
 & \pm(8-k)x^2 \mp 2(8-k)x \pm (8-k)k \\ \hline
 & (2k-9)x-(8-k)k+10
\end{array}
$$

$\therefore$ Remainder = $(2k-9)x-(8-k)k+10$

On comparing this remainder with given remainder, i.e. $x+a$,

$2k-9=1 \Rightarrow 2k=10$

$\Rightarrow\ k=5$

And $-(8-k)k+10=a$

$\Rightarrow\ a=-(8-5)5+10=-5$

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