MATH-O-MANIA

Exercises 2.3 (Polynomials)

1. Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in each of the following:

- **(i)** p(x) = $x^3 - 3x^2 + 5x - 3$; g(x) = $x^2 - 2$
- **(ii)** p(x) = $x^4 - 3x^2 + 4x + 5$; g(x) = $x^2 + 1 - x$
- **(iii)** p(x) = $x^4 - 5x + 6$;g(x) = $2 - x^2$

Solution:

(i) **(i) p(x) = $x^3 - 3x^2 + 5x - 3$; g(x) = $x^2 - 2$**

$$x - 3$$

$$x^2 - 2 \;\big)\; x^3 - 3x^2 + 5x - 3$$

$$\underset{-}{}x^3 \qquad \underset{+}{-}2x$$

$$-3x^2 + 7x - 3$$

$$\underset{+}{-}3x^2 \qquad \underset{-}{+}6$$

$$7x - 9$$

Quotient = x-3 and remainder 7x – 9

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(ii) p(x) = $x^4 - 3x^2 + 4x + 5$; g(x) = x^2 + 1 – x

$$
\begin{array}{r|l}
 & x^2 + x - 3 \\
\hline
x^2 - x + 1 & x^4 \quad - 3x^2 + 4x - 5 \\
 & \underline{x^4 \mp x^3 \pm x^2} \quad (-\ +\ -) \\
 & x^3 - 4x^2 + 4x + 5 \\
 & \underline{x^3 \mp x^2 \pm x} \quad (-\ +\ -) \\
 & -3x^2 + 3x + 5 \\
 & \underline{\mp 3x^2 \pm 3x \mp 3} \quad (+\ -\ +) \\
 & 8
\end{array}
$$

Quotient = x^2 + x – 3 and remainder 8

(iii) p(x) = $x^4 - 5x + 6$;g(x) = 2 – x^2

$$
\begin{array}{r|l}
 & -x^2 - 2 \\
\hline
-x^2 + 2 & x^4 \quad - 5x + 6 \\
 & \underline{x^4 \mp 2x^2} \quad (-\ +) \\
 & 2x^2 - 5x + 6 \\
 & \underline{2x^2 \quad \mp 4} \quad (-\ +) \\
 & -5x + 10
\end{array}
$$

Quotient = $-x^2$ -2 and remainder -5x +10

2. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

Answer

(i) $t^2 - 3,\ 2t^4 + 3t^3 - 2t^2 - 9t - 12$

$$
\begin{array}{r l}
 & 2t^2 + 3t + 4 \\
t^2 - 3 \,\big) & 2t^4 + 3t^3 - 2t^2 - 9t - 12 \\
 & \pm 2t^4 \qquad \mp 6t^2 \\
\hline
 & +3t^3 + 4t^2 - 9t - 12 \\
 & \pm 3t^3 \qquad \mp 9t \\
\hline
 & +4t^2 \qquad - 12 \\
 & \pm 4t^2 \qquad \mp 12 \\
\hline
 & 0
\end{array}
$$

Since on division the remainder is 0, so we can say that the first polynomial is a factor of the second polynomial.

(ii) $x^2 + 3x + 1,\ 3x^4 + 5x^3 - 7x^2 + 2x + 2$

$$
\begin{array}{r l}
 & 3x^2 - 4x + 2 \\
x^2 + 3x + 1 \,\big) & 3x^4 + 5x^3 - 7x^2 + 2x + 2 \\
 & \pm 3x^4 \pm 9x^3 \pm 3x^2 \\
\hline
 & -4x^3 - 10x^2 + 2x + 2 \\
 & \mp 4x^3 \mp 12x^2 \mp 4x \\
\hline
 & +2x^2 + 6x + 2 \\
 & \pm 2x^2 \pm 6x \pm 2 \\
\hline
 & 0
\end{array}
$$

Since on division the remainder is 0, so we can say that the first polynomial is a factor of the second polynomial.

(iii) $x^3 - 3x + 1$, $x^5 - 4x^3 + x^2 + 3x + 1$

$$\begin{array}{r|l} & x^2 - 1 \\ x^3 - 3x + 1 & x^5 - 4x^3 + x^2 + 3x + 1 \\ & \pm x^5 \mp 3x^3 \pm x^2 \\ \hline & -x^3 + 3x + 1 \\ & \mp x^3 \pm 3x \mp 1 \\ \hline & 2 \end{array}$$

Since we are getting a non-zero remainder so we can say that g(x) is not a factor of p(x).

3. Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$ if two of its zeroes are √5/3 and - √5/3 .

Ans. Two zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$ are **√5/3 and - √5/3 .**

which means that $\left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right) = x^2 - \frac{5}{3}$ is a factor of $3x^4 + 6x^3 - 2x^2 - 10x - 5$

Applying Division Algorithm to find more factors we get:

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$$
\begin{array}{r}
3x^2+6x+3 \\
x^2-\frac{5}{3}\,\big)\,3x^4+6x^3-2x^2-10x-5 \\
\pm 3x^4 \qquad \mp 5x^2 \qquad\qquad \\
\hline
+6x^3+3x^2-10x-5 \\
\pm 6x^3 \qquad \mp 10x \qquad \\
\hline
+3x^2 \qquad -5 \\
\pm 3x^2 \qquad \mp 5 \\
\hline
0 \qquad
\end{array}
$$

We have P(x) = g(x)*q(x) + R(x)

R(x) = 0

So ,
$3x^4+6x^3-2x^2-10x-5$ = $(x^2-5/3)$ ($3x^2+6x+3)$

Now we will get the other two factors from ($3x^2+6x+3$), which on factorizing will give us

($3x^2+6x+3$) = 3 (x+1)(x+1)

Therefore, the other two factors are (x+1) = 0 and (x+1) = 0
both on solving will give us x = -1.

4. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find $g(x)$.
Answer
Here in the given question,

Dividend = $x^3 - 3x^2 + x + 2$
Quotient = $x - 2$
Remainder = $-2x + 4$
Divisor = $g(x)$
We know that,

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Dividend = Quotient × Divisor + Remainder

⇒ $x^3 - 3x^2 + x + 2 = (x - 2) \times g(x) + (-2x + 4)$⇒ $x^3 - 3x^2 + x + 2 - (-2x + 4) = (x - 2) \times g(x)$
⇒ $x^3 - 3x^2 + 3x - 2 = (x - 2) \times g(x)$
⇒ $g(x) = (x^3 - 3x^2 + 3x - 2)/(x - 2)$

Quotient: $x^2 - x + 1$

$x - 2$) $x^3 - 3x^2 + 3x - 1$
$x^3 - 2x^2$
$-\quad +$
$-x^2 + 3x - 2$
$-x^2 + 2x$
$+\quad -$
$x - 2$
$x - 2$
$-\quad +$
0

∴ $g(x) = (x^2 - x + 1)$

5.Give examples of polynomial *p*(*x*), *g*(*x*), *q*(*x*) and *r*(*x*), which satisfy the division algorithm and
(i) deg *p*(*x*) = deg *q*(*x*)
(ii) deg *q*(*x*) = deg *r*(*x*)
(iii) deg *r*(*x*) = 0
Answer
(i) Let us assume the division of $6x^2 + 2x + 2$ by 2
Here, $p(x) = 6x^2 + 2x + 2$
$g(x) = 2$
$q(x) = 3x^2 + x + 1$
$r(x) = 0$
Degree of $p(x)$ and $q(x)$ is same i.e. 2.
Checking for division algorithm,
$p(x) = g(x) \times q(x) + r(x)$
Or, $6x^2 + 2x + 2 = 2x\,(3x^2 + x + 1)$
Hence, division algorithm is satisfied.

(ii) Let us assume the division of $x^3 + x$ by x^2,

Here, $p(x) = x^3 + x$

$g(x) = x^2$

$q(x) = x$ and $r(x) = x$

Clearly, the degree of $q(x)$ and $r(x)$ is the same i.e., 1.

Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$x^3 + x = (x^2) \times x + x$

$x^3 + x = x^3 + x$

Thus, the division algorithm is satisfied.

(iii) Let us assume the division of $x^3 + 1$ by x^2.

Here, $p(x) = x^3 + 1$

$g(x) = x^2$

$q(x) = x$ and $r(x) = 1$

Clearly, the degree of $r(x)$ is 0.

Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$x^3 + 1 = (x^2) \times x + 1$

$x^3 + 1 = x^3 + 1$

Thus, the division algorithm is satisfied.

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