Exercise 1.4

1. Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal

expansion:

(i) 13/3125 (ii) 17/8 (iii) 64/455 (iv) 15/1600 (v) 29/343

(vi) 23/23 × 52 (vii) 129/22 × 57 × 75 viii) 6/15 (ix) 35/50 (x) 77/210

Answer

(i) 13/3125

Factorize the denominator we get

3125 =5 × 5 × 5 × 5 × 5 = 5^5

So denominator is in form of 5m so it is terminating .

(ii) 17/8

Factorize the denominator we get

8 =2 × 2 × 2 = 2^3

So denominator is in form of 2m so it is terminating .

(iii) 64/455

Factorize the denominator we get

455 =5 × 7 × 13

There are 7 and 13 also in denominator so denominator is not in form of 2m × 5n . so it is not terminating.

(iv) 15/1600

Factorize the denominator we get

1600 =2 × 2 × 2 ×2 × 2 × 2 × 5 × 5 = $2^6 \times 5^2$

so denominator is in form of 2m × 5n

Hence it is terminating.

(v) 29/343

Factorize the denominator we get

343 = 7 × 7 × 7 = 7^3

There are 7 also in denominator so denominator is not in form of 2m × 5n

Hence it is non-terminating.

(vi) 23/($2^3 \times 5^2$)

Denominator is in form of 2m × 5n

Hence it is terminating.

(vii) 129/($2^2 \times 5^7 \times 7^5$)

Denominator has 7 in denominator so denominator is not in form of 2m × 5n

Hence it is none terminating.

(viii) 6/15

Divide nominator and denominator both by 3 we get 2/5

Denominator is in form of 5m so it is terminating.

(ix) 35/50 divide denominator and nominator both by 5 we get 7/10

Factorize the denominator we get

$10=2 \times 5$

So denominator is in form of $2m \times 5n$ so it is terminating.

(x) 77/210

Simplify it by dividing nominator and denominator both by 7 we get 11/30

Factorize the denominator we get

$30=2 \times 3 \times 5$

Denominator has 3 also in denominator so denominator is not in form of $2m \times 5n$

Hence it is none terminating.

2. Write down the decimal expansions of those rational numbers in Question 1 above which have terminating decimal expansions.

Answer

(i) $13/3125 = 13/55 = 13\times25/55\times25 = 416/105 = 0.00416$

(ii) $17/8 = 17/23 = 17\times53/23\times53 = 17\times53/103 = 2125/103 = 2.125$

(iv) $15/1600 = 15/24\times102 = 15\times54/24\times54\times102 = 9375/106 = 0.009375$

(vi) $23/2352 = 23\times53\times22/23\ 52\times53\times22 = 11500/105 = 0.115$

(viii) $6/15 = 2/5 = 2\times2/5\times2 = 4/10 = 0.4$

(ix) 35/50 = 7/10 = 0.7.

3. The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational, and of the form p , q you say about the prime factors of q?

(i) 43.123456789

(ii) 0.120120012000120000...

(iii) 43.123456789

Answer

(i) Since this number has a terminating decimal expansion, it is a rational number of the form p/q, and q is of the form 2m × 5n.

(ii) The decimal expansion is neither terminating nor recurring. Therefore, the given number is an irrational number.

(iii) Since the decimal expansion is non-terminating recurring, the given number is a rational number of the form p/q, and q is not of the form 2m × 5n.