Exercise 1.3 (Real Numbers)

1. Prove that √5 is irrational.

Answer

Let take √5 as rational number

If a and b are two co prime number and b is not equal to 0.

We can write √5 = a/b

Multiply by b both side we get

b√5 = a

To remove root, Squaring on both sides, we get

$5b^2$ = a^2 -------- (i)

Therefore, 5 divides a^2 and according to theorem of rational number, for any prime number p which is a factor of a^2 then it will divide a also.

That means 5 will divide a also. So we can write

a = 5c

Putting value of a in equation (i) we get

$5b^2$ = $(5c)^2$

$5b^2$ = $25c^2$

Divide by 25 we get

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$\frac{(b)^2}{5} = c^2$

Similarly, we get that b will divide by 5

and we have already get that a is divide by 5

but a and b are co-prime number. So it contradicts.

Hence √5 is not a rational number, it is irrational.

2. Prove that 3 + 2√5 is irrational.

Answer

Let take that 3 + 2√5 is a rational number.

So we can write this number as

3 + 2√5 = a/b

Here a and b are two co prime number and b is not equal to 0

Subtract 3 both sides we get

2√5 = a/b – 3

2√5 = (a-3b)/b

Now divide by 2, we get

√5 = (a-3b)/2b

Here a and b are integer so (a-3b)/2b is a rational number so √5 should be a rational number But √5 is a irrational number so it contradicts.

Hence, 3 + 2√5 is a irrational number.

3. Prove that the following are irrationals:

(i) 1/√2 (ii) 7√5 (iii) 6 + √2

Answer

(i) Let take that $1/\sqrt{2}$ is a rational number.

So we can write this number as

$1/\sqrt{2} = a/b$

Here a and b are two co prime number and b is not equal to 0

Multiply by $\sqrt{2}$ both sides we get

$1 = (a\sqrt{2})/b$

Now multiply by b

$b = a\sqrt{2}$

divide by a we get

$b/a = \sqrt{2}$

Here a and b are integer so b/a is a rational number so $\sqrt{2}$ should be a rational number But $\sqrt{2}$ is a irrational number so it contradicts.

Hence, $1/\sqrt{2}$ is a irrational number

(ii) Let take that $7\sqrt{5}$ is a rational number.

So we can write this number as

$7\sqrt{5} = a/b$

Here a and b are two co prime number and b is not equal to 0

Divide by 7 we get

$\sqrt{5} = a/(7b)$

Here a and b are integer so a/7b is a rational number so $\sqrt{5}$ should be a rational number but $\sqrt{5}$ is a irrational number so it contradicts.

Hence, $7\sqrt{5}$ is a irrational number.

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(iii) Let take that $6 + \sqrt{2}$ is a rational number.

So we can write this number as

$6 + \sqrt{2} = a/b$

Here a and b are two co prime number and b is not equal to 0

Subtract 6 both side we get

$\sqrt{2} = a/b - 6$

$\sqrt{2} = (a-6b)/b$

Here a and b are integer so $(a-6b)/b$ is a rational number so $\sqrt{2}$ should be a rational number.

But $\sqrt{2}$ is a irrational number so it contradicts.

Hence, $6 + \sqrt{2}$ is a irrational number.

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